Asymmetric Orthogonal Frequency Division Multiplexing

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The system described in this paper is an asymmetric adaptation of orthogonal frequency division multiplexing (OFDM). In a conventional OFDM system, the signal processing hardware is divided equally between the base station and the terminal. In the asymmetric system, most of the complex signal processing hardware is shifted to the base station, making the terminal a simpler and more power-efficient device. To send information to the base station, the terminal transmits a series of QPSK symbols that make up an OFDM-code. The code is designed to distribute the signal's energy into a number of OFDM sub-carriers which can be detected and combined within the base station's OFDM receiver. Other users transmit the same OFDM-codes within the same bandwidth and at the same time, but with slightly offset carrier frequencies. Because of the nature of OFDM, the codes from different users remain orthogonal, even with multipath dispersion. OFDM signals transmitted from the base station are detected at the terminal using a decimator-accumulator structure.

1. Introduction

Asymmetric orthogonal frequency division multiplexing is part of a more general research project on asymmetric radio systems [1]. In this context, the word *asymmetric* refers to the hardware implementation. When designing an asymmetric system, most of the complex hardware – especially the digital signal processing – is implemented in the base station and the terminal is designed to be as simple, inexpensive, and energy-efficient as possible.

Asymmetric structures also have the advantage of versatility. Suppose, for example, that there are two different asymmetric systems: one designed for a local area network, and another designed for a longer-range PCS network. Both are configured so that the signal processing hardware is in the base station. Since neither network requires any complex hardware in the terminal, both networks can use the same terminal structure. The user is then able to roam from the local area network to the PCS network or vice versa without having to change hardware. The base station structure for high-speed (160Mbit/s) local area networks was described in a previous paper [2]. The system described below is applicable to longer-range PCS networks.

This system is based on the principles of orthogonal frequency division multiplexing (OFDM) but also has similarities to code division multiple access (CDMA) systems. Like a CDMA system, each terminal transmits information to the base station on a code made up of a sequence of chips, and all of the terminals transmit their codes simultaneously in the same frequency band. However, the OFDM-codes used in this system are unlike the codes used in CDMA systems. They are not chosen for their autocorrelation properties – in fact OFDM-codes have very poor autocorrelation properties – but are selected so that their frequency spectra resemble the spectra of OFDM signals. This allows the codes from different terminals to be separated at the base station using an OFDM receiver without the co-user interference normally associated with CDMA. Even when the multipath radio channel is dispersive, the signals from different users remain orthogonal.

Figure 1 shows the simplified structure of an asymmetric-OFDM system. The base station contains a conventional OFDM transceiver (described in detail in Section 2) which uses a time division duplexed transmitter and receiver. The terminal's transmitter is similar to a CDMA transmitter with a high-speed code (described in Section 4) used to carry a slower-speed data sequence. Both the data sequence and the code are made up of QPSK symbols. The terminal's receiver uses two devices to decode the OFDM signals from the base station. The accumulator ($\sum x_i$) is used to decode the basic OFDM signals described in Section 3. The decimator ($\downarrow M$) is added to decode the more complex signals with diversity as described in Section 5. Note that only the base station requires a complete OFDM transceiver with all the associated signal processing hardware. The components in the terminal are trivial and require little hardware to implement.

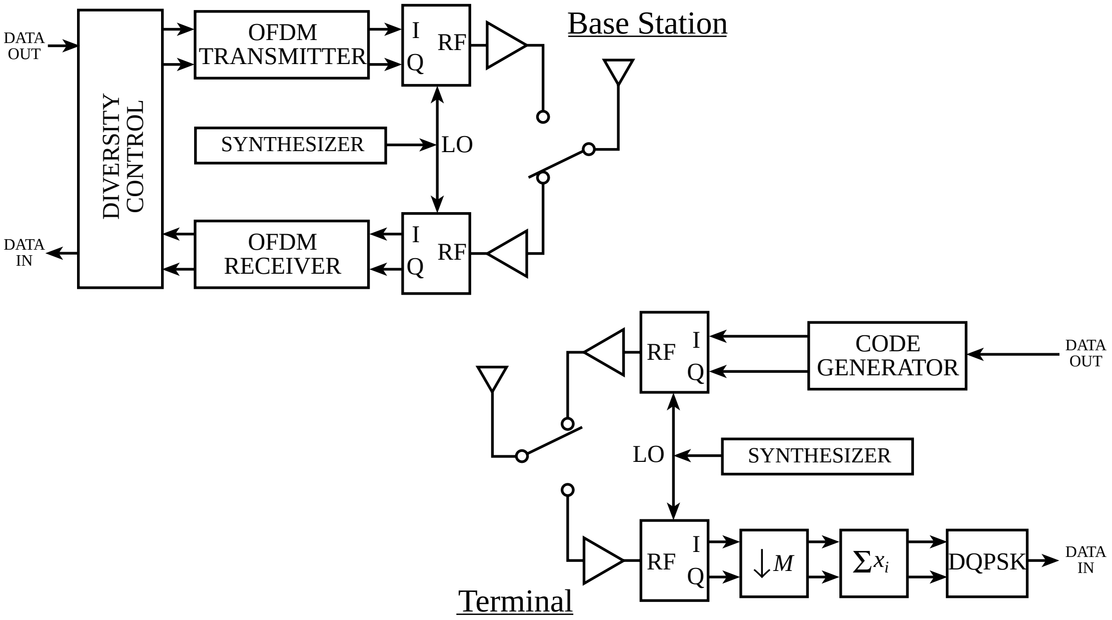


Figure 1 - Asymmetric OFDM structure.

2. Orthogonal Frequency Division Multiplexing

Before introducing asymmetric-OFDM, the principles of conventional orthogonal frequency division multiplexing are reviewed. OFDM was originally proposed by Chang in 1966 [3] and then improved with fast Fourier transform techniques by Weinstein and Ebert in 1971 [4]. It works by transforming a wideband radio channel into a set of narrowband channels, possibly hundreds or even thousands. Each of the OFDM sub-channels is capable of carrying only a fraction of the information of the wideband radio signal, but because there are many of them operating in parallel the aggregate data rate can be just as high.

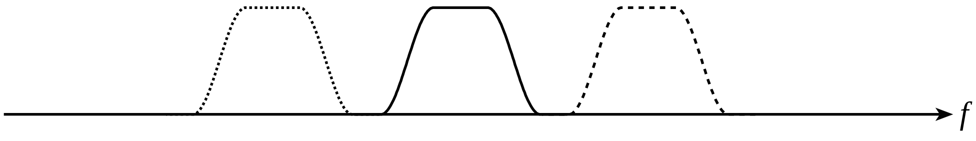
The key feature of OFDM modulation is the efficiency in which it packs together very narrow channels in the frequency domain. A standard frequency division multiplexing (FDM) spectrum is shown in Figure 2a. The different channels are filtered to a narrow bandwidth and then modulated to different frequencies by a set of sub-carriers. To keep the signals independent, guard bands are inserted between the sub-channels so that the receiver can filter out individual sub-channels for decoding. While effective, FDM is not a very efficient use of radio spectrum, mainly because of the waste caused by the guard bands.

Figure 2b shows the spectra of three OFDM sub-carriers defined by

$$x_0(n) = \begin{cases} 1 & 0 \le n < N \\ 0 & \text{otherwise} \end{cases} \tag{2.1}$$

$$x_1(n) = \begin{cases} e^{j2\pi n/N} & 0 \le n < N \\ 0 & \text{otherwise} \end{cases} \tag{2.2}$$

a) Frequency Division Multiplexing



b) Orthogonal Frequency Division Multiplexing

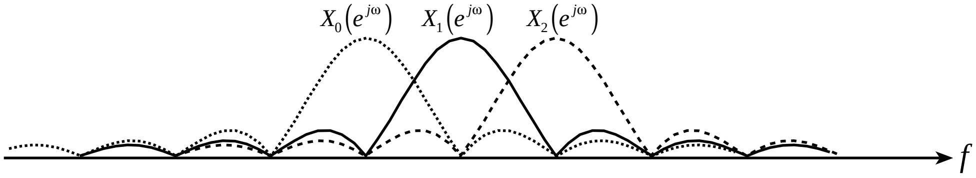


Figure 2 - Comparison between the spectra of (a) frequency division multiplexed signals and (b) orthogonal frequency division multiplexed signals.

$$x_2(n) = \begin{cases} e^{j4\pi n/N} & 0 \le n < N \\ 0 & \text{otherwise} \end{cases}. \qquad 2.3$$

The zeroth sub-carrier $x_0(n)$ is a rectangular pulse N samples long, sub-carrier one $x_1(n)$ is one cycle of a complex sinusoid, and sub-carrier two $x_2(n)$ is two cycles of a complex sinusoid. Note that between the spectra of these three sub-carriers, there are no guard bands and the signals are packed so closely in frequency that the spectra actually overlap. It is the specific shape of the signal spectra that makes this possible. Each of the three OFDM signals has a spectrum with one main lobe and N-1 zeroes spaced at intervals of $2\pi / N$ radians around the unit circle. By positioning the signals as in Figure 2b, the main lobe of each sub-carrier lines up with the zeros of all the other nodes. Therefore at those frequencies, there is no interference between sub-carriers and they are perfectly orthogonal.

Using this method, it is possible to pack up to N independent sub-carriers around the unit circle and modulate a different piece of information on each one. Sub-carrier k with information symbol m_k is expressed as

$$x_k(n) = \begin{cases} m_k\, e^{j2\pi kn/N} & 0 \le n < N \\ 0 & \text{otherwise} \end{cases}. \qquad 2.4$$

When all the sub-carriers are combined together, the result is

$$x(n) = \sum_{k=0}^{N-1} x_k(n) \qquad 2.5$$

$$x(n) = \begin{cases} \sum_{k=0}^{N-1} m_k\, e^{j2\pi kn/N} & 0 \le n < N \\ 0 & \text{otherwise} \end{cases} \qquad 2.6$$

$$x(n) = \begin{cases} N\left[\frac{1}{N}\sum_{k=0}^{N-1} m_k\, W_N^{-kn}\right] & 0 \le n < N \\ 0 & \text{otherwise} \end{cases}. \qquad 2.7$$

$$\text{where} \quad W_N = e^{-j2\pi/N}$$

In the final form of the OFDM signal (Equation 2.7) the part within the brackets is the inverse discrete Fourier transform of the information sequence. Therefore, the OFDM signal is calculated by applying an inverse-DFT to the set of message symbols and multiplying by N.

In order to recover the information, the receiver needs to calculate the spectrum of the OFDM signal at the *N* points around the unit circle where the sub-carriers are orthogonal. A discrete Fourier transform of length *N* directly returns exactly these frequency domain samples, therefore the information can be recovered by performing a DFT on the received signal. Both the inverse-DFT in the transmitter and the DFT in the receiver are implemented using the fast Fourier transform and *N* is usually a power of two.

3. Asymmetric-OFDM

The conventional OFDM method described above is not asymmetric. Both the transmitter and receiver require roughly an equal amount of signal processing hardware. To make OFDM asymmetric, it should be adapted to operate within the limited signal processing capabilities of the terminal described in Section 1. Both the inverse-DFT in the transmitter and the DFT in the receiver require signal processing resources that the terminal does not have. There is, however, one sub-carrier that the terminal can access, and this is the basis of asymmetric-OFDM. The discrete Fourier transform of the zeroth sub-carrier is given by

$$X_0 = \sum_{n=0}^{N-1} x(n) W_N^0 = \sum_{n=0}^{N-1} x(n), \qquad 3.1$$

which is simply the sum of *N* samples. The terminal's accumulator is used to compute Equation 3.1 and recover the zeroth sub-carrier from an OFDM signal. Generating the zeroth sub-carrier is also a trivial operation given by

$$x_0(n) = \begin{cases} m_0 & 0 \le n < N \\ 0 & \text{otherwise} \end{cases}, \qquad 3.2$$

which is the message symbol repeated for *N* samples.

An OFDM system that only uses one sub-carrier is not very useful. Suppose, however, that the base station transmits an OFDM signal with *N* sub-carriers, where each sub-carrier is destined for a different terminal. Each terminal then only needs to calculate one point of the DFT to recover its particular sub-carrier. The terminal using sub-carrier zero demodulates its part of the signal with its accumulator using the method described above. The terminal assigned to sub-carrier one must compute

$$X_1 = \sum_{n=0}^{N-1} x(n) W_N^n, \qquad 3.3$$

however, this requires multiplying each sample by W_N^n before accumulating, which is also beyond the terminal's abilities. It avoids this unwanted multiplication by slightly adjusting its radio frequency synthesizer (Figure 1) so that the base station's signal is shifted down in frequency by an amount equal to the spacing between sub-carriers. Now sub-carrier one is shifted down to the position where sub-carrier zero used to be, and the terminal can detect it using its accumulator. Similarly, all the other terminals adjust their RF synthesizers so that their respective sub-carriers appear at OFDM position zero. In this way, each terminal detects a different OFDM sub-carrier using the same method.

The same technique works for transmitting OFDM signals back to the base station. Each terminal simultaneously generates and transmits the zeroth OFDM sub-carrier described by Equation 3.2. However, since each terminal is tuned to a different sub-carrier frequency, the base station receives N sub-carriers at different frequencies, which it can recover using its OFDM receiver.

Changing the number of sub-carriers, N, in an asymmetric-OFDM system changes the effective range of the terminal without having to change its output power or the bandwidth of its filters. Suppose each sample (chip) has a fixed energy E_0. Each symbol (Equation 3.2) is composed of N samples and has an energy of NE_0. It is not unusual to have a thousand or more sub-carriers in an OFDM system, which would increase the signal's energy proportionally and give the terminal the potential of transmitting signals a very long distance. Theoretically, it should be possible to increase the number of OFDM sub-carriers and the symbol energy arbitrarily to transmit signals any distance. In practice OFDM packets need to be much shorter than the coherence time of the channel, therefore the Doppler effect limits how long a packet can be and determines the system's maximum range.

The compromise for using asymmetric-OFDM is that the maximum data rate available to any one terminal is reduced. While the overall capacity of the base station does not change with different values of N, OFDM divides the bandwidth into N individual sub-carriers. Since a simple terminal can only access one sub-carrier at a time, its maximum data rate is roughly $1/N$ times the full capacity of the base station.

4. Asymmetric-OFDM with Frequency Diversity

The system described above allows each terminal to generate one OFDM sub-carrier that can be decoded at the base station. However, since individual sub-carriers are narrowband signals and subject to flat fading, the signal from any given terminal may be lost if its sub-carrier frequency falls at a deep multipath fade. A better system would assign each terminal several sub-carriers within the frequency band so that the terminal can spread its signal energy across different frequencies and avoid the flat fading problem. The multiple sub-carriers all carry the same information and are spaced sufficiently in frequency so that they can be combined within the base station using normal diversity combining techniques.

The terminal does not have an OFDM transmitter to generate multiple sub-carriers, however it can generate sequences of chips using its code generator (Figure 1). The *OFDM-codes* derived below are sequences of symbols whose spectra contain multiple OFDM sub-carriers. Each chip of the code is limited to one of the four QPSK constellation states (1, *j*, –1, and –*j*) to keep the code generator simple. The transmitter in the terminal multiplies each chip in the code by a QPSK symbol that represents the data to be transmitted. The base station decodes each of the sub-carriers generated by the code using its OFDM receiver and combines them into a single signal using maximal ratio combining [5]. Now if a single sub-carrier is lost to a multipath fade, the base station can still recover the signal from the remaining sub-carriers.

To construct an OFDM-code of length *N* with *R* sub-carriers, start with the zeroth sub-carrier of length *N*/*R* as defined by Equation 3.2. For convenience, the signal is represented by its *z*-transform

$$X_0(z) = m \sum_{n=0}^{N/R-1} z^{-n} \,. \tag{4.1}$$

The signal defined by Equation 4.1 contains *N*/*R* sub-carriers with only the one at the zeroth position occupied by message *m*. To create a signal with *R* sub-carriers, replace the *z* in Equation 4.1 with z^R to produce the following new signal

$$X(z) = X_0\left(z^R\right) = m \sum_{n=0}^{N/R-1} z^{-Rn} \,. \tag{4.2}$$

What this does is the multirate signal processing operation of *expansion* [6] which has the effect of creating *R* copies of the original spectrum in the frequency domain. Now there is a total of *N* sub-carriers with *R* of them occupied. For example, the coefficients of $X(z)$ for *N*=64 and *R*=4 are

$$\begin{aligned} x(n) = \{&m,0,0,0,m,0,0,0,m,0,0,0,m,0,0,0,\\ &m,0,0,0,m,0,0,0,m,0,0,0,m,0,0,0,\\ &m,0,0,0,m,0,0,0,m,0,0,0,m,0,0,0,\\ &m,0,0,0,m,0,0,0,m,0,0,0,m,0,0,0\} \end{aligned} \tag{4.3}$$

and the spectrum of the signal is shown in Figure 3. As expected, there are four occupied sub-carriers and sixty vacant ones for other terminals to use.

The problem with the method above is power. OFDM signals get their processing gain by repeating the same information over again for many samples. In the code defined by Equation 4.3, three out every four samples are zero and the power of the signal actually drops by three-quarters over the no-diversity case. This drop in power can be avoided by using a *base code* of length *R*, as defined by

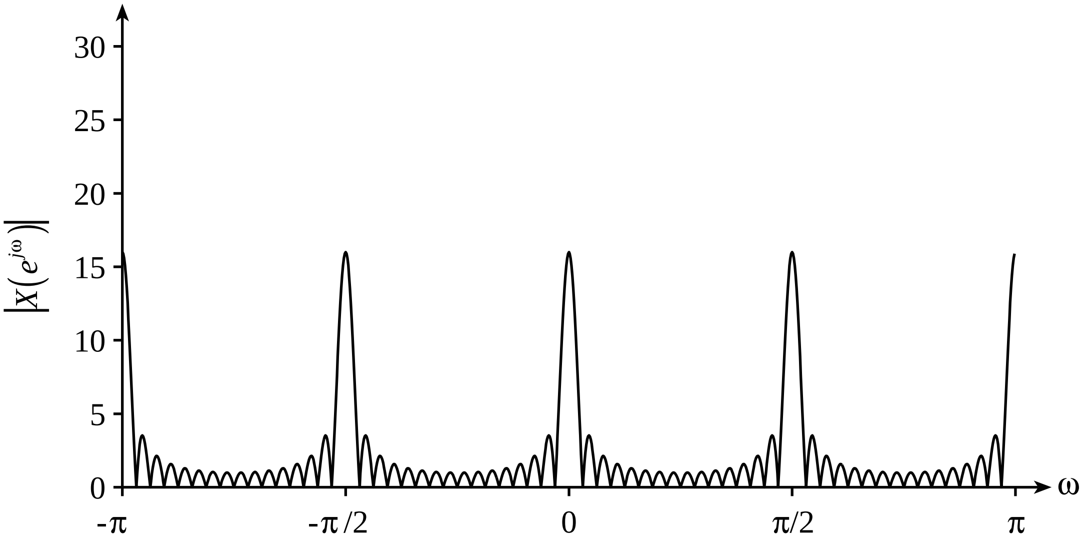


Figure 3 - Frequency spectrum of the code given in Equation 4.3.

$$B(z) = \sum_{n=0}^{R-1} b(n) z^{-n}, \qquad 4.4$$

where the coefficients $b(n)$ belong to the set {1, j, -1, -j}. When the base code polynomial is multiplied by the polynomial defined in Equation 4.2, the base code fills in the empty space in the OFDM signal to produce the following

$$X_C(z) = X(z)B(z) = m\left[\sum_{n=0}^{N/R-1} z^{-Rn}\right]\left[\sum_{p=0}^{R-1} b(p) z^{-p}\right] = m \sum_{n=0}^{N/R-1} \sum_{p=0}^{R-1} b(p) z^{-(Rn+p)}. \qquad 4.5$$

The coefficients of the OFDM-code $x_C(n)$ are the base code coefficients $b(n)$ multiplied by the message symbol m and repeated N/R times. The spectrum of the OFDM-code is a product of the spectra of the OFDM signal $X(e^{j\omega})$ and the base code $B(e^{j\omega})$ and is shown in Figure 4 (for N=32, R=4). The OFDM signal spectrum $X(e^{j\omega})$ defines the shape of the OFDM-code spectrum with its R occupied sub-carriers and N-R null sub-carriers. The base code spectrum $B(e^{j\omega})$ amplifies the power of the occupied sub-carriers. In most cases it is desirable to boost the magnitude of all of the diversity carriers by the same amount, therefore the base code coefficients are chosen so that the magnitude of $B(e^{j\omega})$ is the same at the frequency of each of the occupied sub-carriers.

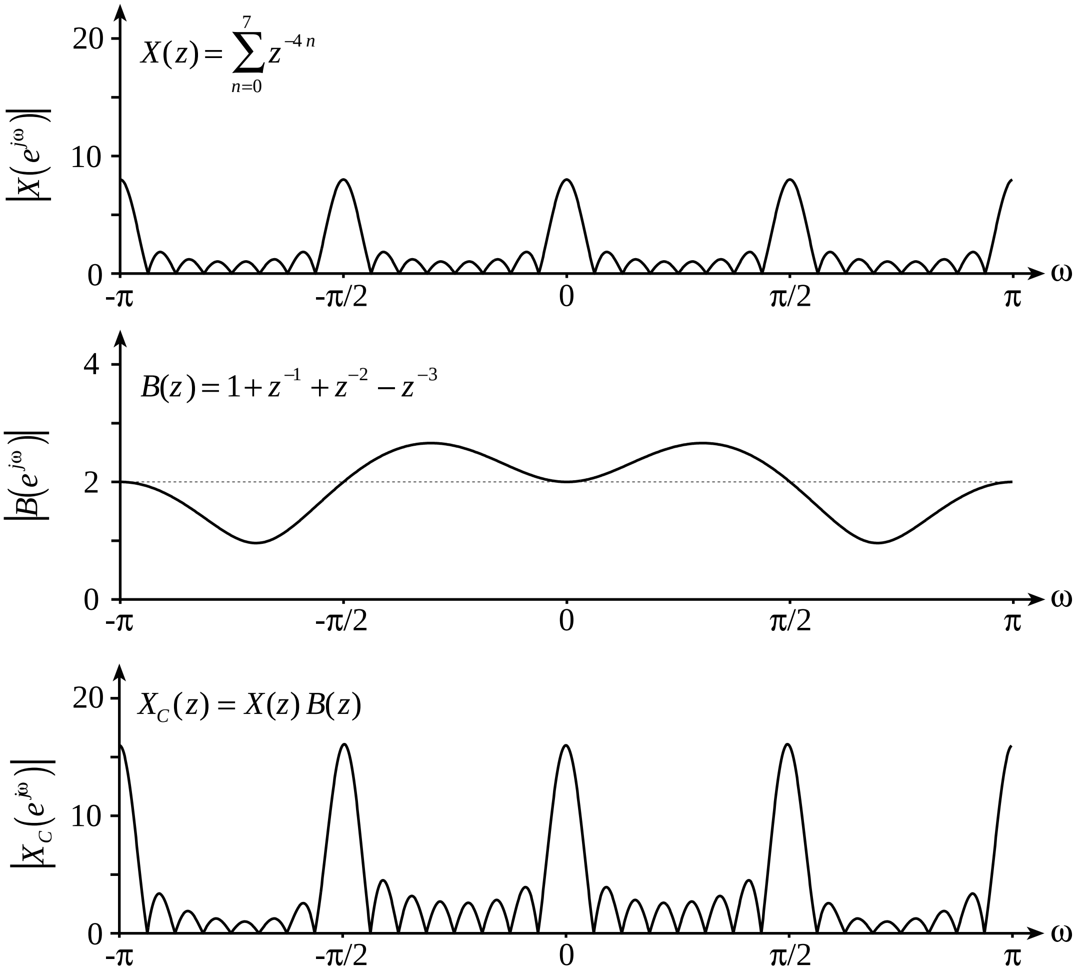


Figure 4 - Frequency spectra of the components of an OFDM-code: (a) the OFDM signal, (b) the base code, and (c) the product of the OFDM signal and the base code.

An example OFDM-code is derived for N=64 and R=4 as follows. Since the result has four sub-carriers at frequencies 0, $\pi/2$, π, and $-\pi/2$, the base code should have equal amplitude at each of these frequencies, i.e.

$$\left|B\left(e^{j0}\right)\right|=\left|B\left(e^{j\pi/2}\right)\right|=\left|B\left(e^{j\pi}\right)\right|=\left|B\left(e^{-j\pi/2}\right)\right|. \qquad 4.6$$

The length four Frank-Heimiller code [7] given by

$$b(n)=\{1,1,1,-1\} \qquad 4.7$$

meets this specifications and has a constant magnitude of two at the sub-carrier frequencies. The OFDM-code is evaluated from Equation 4.5 to return the following coefficients

$$\begin{aligned} x_C(n) = \{&m, m, m, -m, m, m, m, -m, m, m, m, -m, m, m, m, -m, \\ &m, m, m, -m, m, m, m, -m, m, m, m, -m, m, m, m, -m, \\ &m, m, m, -m, m, m, m, -m, m, m, m, -m, m, m, m, -m, \\ &m, m, m, -m, m, m, m, -m, m, m, m, -m, m, m, m, -m\} \end{aligned}. \qquad 4.8$$

Figure 5 shows the frequency spectrum of this signal. The result is an OFDM signal with a total of 64 sub-carriers and, as predicted, four of the sub-carriers are occupied with equal amplitude signals and the remainder are empty and available for use by other terminals. The spectral shape of the signal in Figure 5 is identical to that in Figure 3, however the code that generated it has four times as much energy. This is the same energy level as a length-64 OFDM signal without frequency diversity, therefore there is no power penalty for using diversity with these OFDM-codes.

Table 1 shows base codes for one, two, four, eight, and sixteen-way diversity. The four and sixteen base codes are Frank-Heimiller codes. The two and eight length codes were found by a search algorithm. To generate an OFDM-code with *N* total sub-carriers and *R* active sub-carriers, choose the base sequence of length *R*, repeat it a total of *N*/*R* times, and multiply the result by the message symbol *m*.

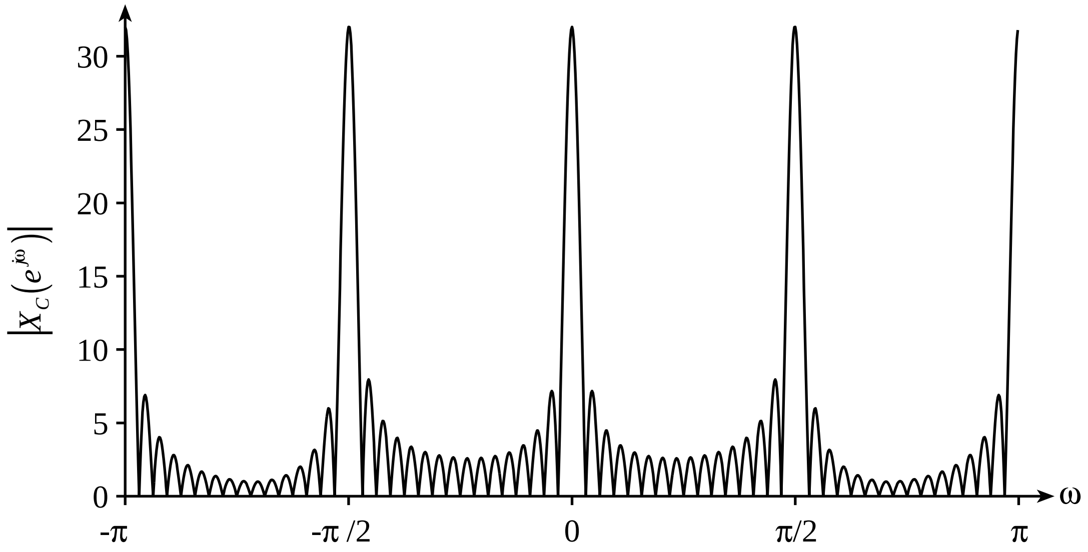


Figure 5 - Frequency spectrum of the code given in Equation 4.8.

Table 1 Base codes for different levels of diversity (*R*)

R	Base Code $b(n)$
1	{ 1 }
2	{ 1, j }
4	{ 1, 1, 1, -1 }
8	{ 1, 1, 1, j, -1, 1, -1, j }
16	{ 1, 1, 1, 1, 1, j, -1, -j, 1, -1, 1, -1, 1, -j, -1, j }

5. Decoding Asymmetric-OFDM at the Terminal

Using OFDM-codes, the simple terminal is able to generate multiple sub-carriers for the base station without having an OFDM transmitter of its own. The terminal must also be able to receive and combine multiple sub-carriers from the base station without an OFDM receiver. It does this using the decimator-accumulator structure shown in Figure1.

The decimator (↓*M*) is a simple device that passes through every *M*th sample and discards the rest [6]. For a given input $x(n)$, the decimated output $y(n)$ is given by

$$y(n) = x(Mn). \qquad 5.1$$

The frequency spectrum of the decimated sequence is given by

$$Y\left(e^{j\omega}\right) = \left[X\left(e^{j\omega}\right)\right]_{\downarrow M} = \frac{1}{M}\sum_{\ell=0}^{M-1} X\left(e^{j(\omega-2\pi\ell)/M}\right). \qquad 5.2$$

Notice that the new frequency spectrum is made up of M frequency-scaled and frequency-shifted copies of the original spectrum. This property makes decimation an ideal operation for recombining sub-carriers within the terminal.

Say that the base station simultaneously transmits the terminal’s message on the four sub-carriers shown in bold in Figure 6. Note that sub-carriers are shown as vertical lines in this figure for clarity and are numbered according to their DFT index. The sub-carriers intended for this terminal are located at indices 0, 16, 32, and 48. After the terminal samples the signal, it decimates it by four. Decimation causes frequency spectrum to be divided into four parts, spread out, and frequency shifted as defined by Equation 5.2. The four parts then combine to create a new spectrum with only 16 sub-carriers. The desired four sub-carriers, marked in bold, all combine into the zeroth sub-carrier. The terminal then uses its accumulator to extract the combined zeroth sub-carrier from the rest of the signal.

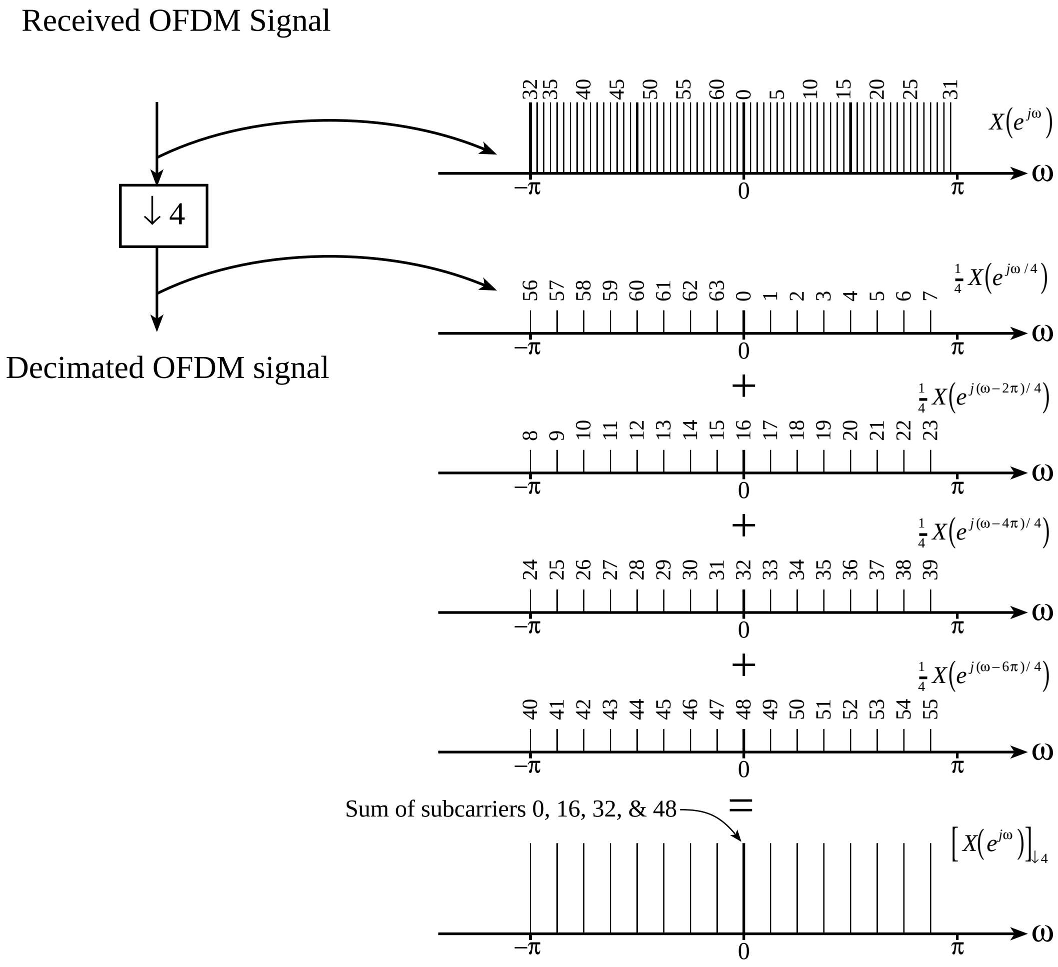


Figure 6 - Demonstration of the use of decimation to combine sub-carriers within an OFDM signal.

The multipath radio channel between the base station and terminal will impose an arbitrary amplitude and phase shift on each of the sub-carriers. The terminal can combine sub-carriers with its decimator, however it can not correct for the effects of the channel phase shift. Left uncorrected, the channel phase shift may cause the sub-carriers to add destructively rather than constructively. The solution to this problem is to have the base station pre-adjust the phase on each sub-carrier so that they arrive at the terminal in-phase.

When the terminal is transmitting its OFDM-codes to the base station, the base station observes the signal coming in from each individual sub-carrier and compares it to the expected value. From this information, it builds a table of the channel phase shift and attenuation for each sub-carrier, which is used internally for the tap weights of the maximal ratio combiner. Since the return signal to the terminal is passed through the same channels using time division duplexing, the table will also be accurate for the downlink. Before transmitting information back to the terminals, the base station adjusts the phases of the sub-carriers so that they combine constructively within the terminals' decimators.

6. Conclusions

An OFDM system has been presented that has most of the hardware in the base station and little hardware in the terminal. The system draws on the characteristics of both OFDM and CDMA systems: using CDMA-like spreading codes to send information from the terminal, but using an OFDM receiver in the base station to interpret these codes. The result is a very simple terminal transmitter that does not require an OFDM modulator, and a base station receiver that can separate the signals without co-user interference. The system is also designed to allow the terminal to receive complex OFDM signals from the base station with a simple decimator-accumulator structure.

Acknowledgements

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References

[1] G.H. McGibney, "Wireless Networking with Simple Terminals", Ph.D. Thesis, The University of Calgay, May 2000.

[2] G.H. McGibney and S.T. Nichols, "An Asymmetrical Implementation of a High Speed Wireless LAN", Wireless 97 Conference, pp.432-443, July 9-11, 1997.

[3] R.W. Chang, "Synthesis of Band-Limited Orthogonal Signals for Multi-Channel Data Transmission", Bell Systems Technical Journal, vol. 45, pp. 1775-1796, Dec. 1966.

[4] S.B. Weinstein and P.M. Ebert, "Data Transmission by Frequency-Division Multiplexing Using the Discrete Fourier Transform", IEEE Transactions on Communication Technology, vol. COM-19, no. 5, Oct. 1971.

[5] D.G. Brennan, "Linear Diversity Combining Techniques", Proceedings of the IRE, vol. 47, pp. 1075-1102, June 1959.

[6] P.P. Vaidyanathan, *Multirate Systems and Filter Banks*, Prentice Hall, Englewood Cliffs, New Jersey, 1993, pp. 100-105.

[7] R.C. Heimiller, "Phase Shift Pulse Codes with Good Periodic Correlation Properties", IRE Transactions on Information Theory, vol. IT-7, no. 4, pp. 254-257, October 1961.